

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 8

If n is negative integer, then $n!$ is not defined. We state binomial theorem in another form

$$(a+b)^n = a^n + \frac{n}{1!} a^{n-1} b + \frac{n(n-1)}{2!} a^{n-2} b^2 + \frac{n(n-1)(n-2)}{3!} a^{n-3} b^3 + \dots + \frac{n(n-1)(n-2)\dots(n-r+1)}{r!} a^{n-r} b^r + \dots + b^n$$

Here, $T_{r+1} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!} a^{n-r} b^r$

The general term of an expansion $(a+b)^n$ is

$$T_{r+1} = {}^nC_r a^{n-r} b^r, 0 \leq r \leq n, r \in N$$

Middle Terms:

1. In $(a+b)^n$, if n is even, then the no. of terms in the expansion is odd. Therefore, there is only one middle term and it is $\left(\frac{n+2}{2}\right)^{th}$ term.

2. In $(a+b)^n$, if n is odd then the no. of terms in the expansion is even. Therefore, there are two middle terms and those are

$$\left(\frac{n+1}{2}\right)^{th} \text{ and } \left(\frac{n+3}{2}\right)^{th} \text{ terms.}$$

In the expansion of $(a+b)^n$,

(i) Taking $a=x$ and $b=-y$, we obtain

$$(x-y)^n = {}^nC_0 x^n - {}^nC_1 x^{n-1} y + {}^nC_2 x^{n-2} y^2 - \dots + (-1)^n {}^nC_n y^n$$

(ii) Taking $a=1$, $b=x$, we obtain

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$$

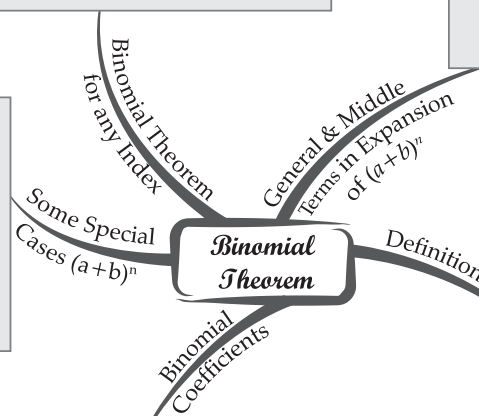
(iii) Taking $a=1$, $b=-x$, we obtain

$$(1-x)^n = {}^nC_0 - {}^nC_1 x + {}^nC_2 x^2 - \dots + (-1)^n {}^nC_n x^n$$

The coefficient ${}^nC_0, {}^nC_1, {}^nC_2, \dots, {}^nC_n$ in the expansion of $(a+b)^n$ are called binomial coefficients and denoted by $C_0, C_1, C_2, \dots, C_n$ respectively

Properties of binomial coefficients:

- $C_0 + C_1 + C_2 + \dots + C_n = 2^n$
- $C_0 - C_1 + C_2 - \dots + (-1)^n C_n = 0$
- $C_0 + C_2 + C_4 + \dots = C_1 + C_3 + C_5 + \dots = 2^{n-1}$
- ${}^nC_r = {}^nC_{n-r} \Rightarrow r_1 = r_2 \text{ or } r_1 + r_2 = n$
- ${}^nC_r + {}^nC_{r-1} = {}^nC_r$
- ${}^nC_r = \frac{n}{r} {}^{n-1}C_{r-1}$



If $a, b \in R$ and $n \in N$, then

$$(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_n a^0 b^n$$

- Remarks:** If the index of the binomial is n then the expansion contains $n+1$ terms.
- In each term, the sum of indices of a and b is always n .
- Coefficients of the terms in binomial expansion equidistant from both the ends are equal.

$$(a-b)^n = {}^nC_0 a^n b^0 - {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 - \dots + (-1)^n {}^nC_n a^0 b^n$$

